Online packing with multiple predictions

proposal for a PhD

February 6, 2025

Supervisor Christoph Dürr, researcher at CNRS, Sorbonne Université, LIP6

Elevator speech

Online algorithms need to process a sequence of requests not knowing the future. They can be augmented by machine learning techniques to predict the future. An active research community focuses on the opportunistic yet prudent usage of these predictions. In this thesis we will investigate combinatorial optimization packing problems in the presence of multiple predictions.

1 Scientific context

In the online algorithm paradigm, the input is revealed to the algorithm in form of a request sequence. Each request must be served without knowing the future requests. Decisions have to satisfy some constraints and they generate some cost. The performance of an online algorithm is measured by the means of the *competitive ratio*, which compares for the worst instance, the optimal offline serving cost with the serving cost generated by the algorithm. In that sense is can be seen as the *price of lack of foresight*. Mainly since 2018, researchers started to investigate on the question of how algorithms could benefit from predictions on the future requests [2]. Indeed recent advances in machine learning allows for predictions on future data based on statistics over the past, which algorithms then can use to take their decisions. A toy example is binary search in an ordered table, which can be solved with $\Theta(\log \eta)$ queries, where η is the error of the predicted insertion index. This performance is ideal, in the sense that it is constant, for constant error, degrades smoothly with the error, without ever exceeding the optimal worst case query complexity ignoring the prediction.

One possible critics to the prediction model is that future requests might depend on a hidden variable, which cannot be learned. For this purpose a model with multiple prediction was introduced, and mostly studied for covering problems. In this thesis we would like to consider multiple predictions for various packing problems. The most studied packing problem is without doubt *bin packing*, which is the main problem the operation research team at Google encounters. Another problem we would like to study is *call admission* or *call control*, where we are given a graph representing a telecommunication network with edge capacities. Requests come in the form of vertex pairs, which if accepted, need to be connected by a path. Bandwidth usage is deduces from the edge capacities along the path. Some variants have been studied in the last year for a single stochastic prediction. The third problem we would like to understand is *lot sizing*. Ever request is a pair (t, d), where t is a requested delivery time and d the size of the demand. Usually the demands need to respect a fixed minimum demand size $d_{\min}$. A demand can be satisfied from the inventory or by placing an order at the delivery time.

There is a setup cost K for each order, independent on the size of the order. In addition there is a storage cost h per item unit and per time unit, also known as a *holding cost*. Its competitive ratio is unbounded, hence this problem is well suited to be studied in the context of predictions.

2 Scientific approach and timeline

Our first step in this thesis will be to study the interval selection problem under multiple predictions, the particular case of call admission on a line graph. Naturally we aim to combine the algorithm from [1] with the multiplicative update technique for the expert problem. Once we understand this problem, we consider its generalization on a cycle. There, a request consists of a pair of vertices, which can be connected via the two possible paths on the cycle. We might need also to consider a larger than unit capacity on the edges to make this problem interesting. And finally our goal is to tackle the general call admission problem on trees, and moreover on general graphs. If we manage to solve these problems, we should be able to produce a good publication, aiming venues such as NeurIPS, IJCAI or AAAI. Depending on how well it goes, this first milestone could reasonably be reached in the first 6 months of the PhD. A journal version could be submitted to the *Journal of Artificial Intelligence Research*.

The second milestone would be to study two other problems in this multiple prediction framework: *Bin packing* and *lot sizing*. The multiple prediction model is well motivated for these problems, as items could arrive from different providers. The distribution generated by each provider can be learned, but the proportions between the providers can change over time in a non-predictable manner. We have informal industrial partners (Orano, Califrais) who could provide us with real data for experiments and validate our models.

In addition to these technical results we think that there is some important meta-observation to be extracted from the bibliography. The design of an online algorithm is typically the compromise between different objectives. This is very visible in linear search or the rent-or-buy problem, but also present in the work function algorithm for metrical task systems. Combining different algorithms in an algorithm design to inherit several good behaviors is a general technique in algorithm design, and used in all on-line algorithms. Such technique is particular important when a compromize is to be found between trusting the predictions and optimizing for the worst case scenario.

3 Contribution to the Sorbonne Center for AI

This PhD will enrich the mathematical foundations of decision making, and contribute to the area of machine learning augmented algorithms.

References

[1] Joan Boyar et al. "Online Interval Scheduling with Predictions". en. In: *Algorithms and Data Structures*. Ed. by Pat Morin and Subhash Suri. Vol. 14079. Series Title: Lecture Notes in Computer Science. Cham: Springer Nature Switzerland, 2023, pp. 193–207. URL: https://link.springer.com/10.1007/978-3-031-38906-1_14 (visited on 01/31/2025).

[2] Alexander Lindermeyer and Nicole Megow. *Algorithms with Predictions*. URL: https://algorithms-with-predictions.github.io/.